

# Sheet 2

Solve the following

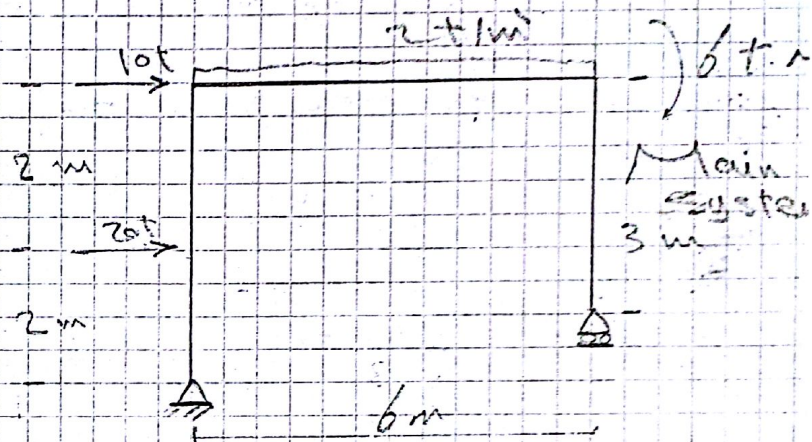
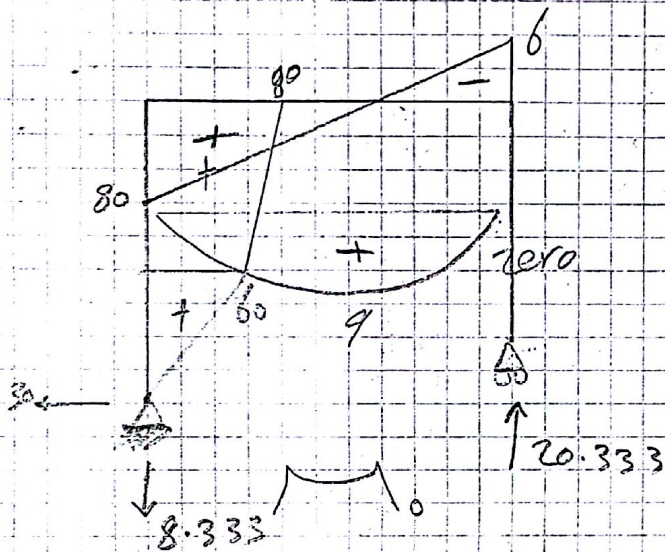
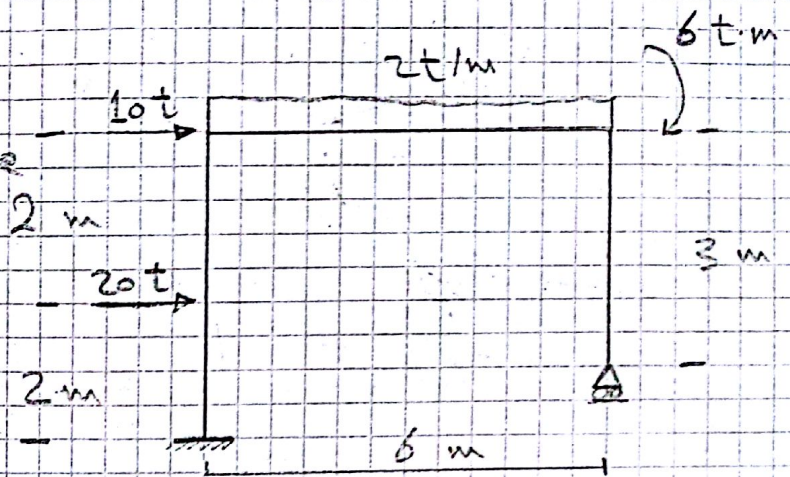
1. Shown loads

②  $40^{\circ}\text{C}$  inside  $20^{\circ}\text{C}$  outside

Unknown = 4

Equations = 3

1<sup>st</sup> indeterminate

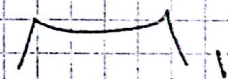
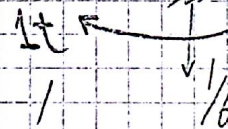


$$\alpha_{10} = \int \frac{M_1 M_0}{EI} dl$$

$$\alpha_{10} = \left[ \frac{1}{2} \times 60 \times 2 + \frac{1}{2} \times 140 \times 2 + \frac{6}{6} \times [2 \times 80 - 2 \times 6 + 80 - 6] + \frac{2}{3} \times 6 \times 9 \right] \frac{1}{EI} = \frac{458}{20000} = 0.0229$$

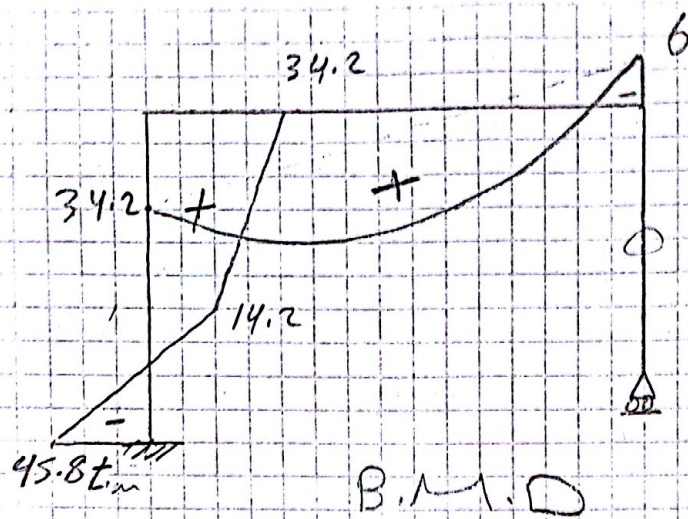
$$\alpha_{11} = \int \frac{M_1 M_1}{EI} dl$$

$$= \frac{1}{EI} (4 + 6) = \frac{10}{20000} = \frac{1}{2000}$$



$$X_1 = \frac{-\alpha_{10}}{\alpha_{11}} = \frac{-0.0229}{5 \times 10^{-4}} = -45.8\text{ t}\cdot\text{m} = 45.8\text{ t}\cdot\text{m}$$





Due to Rise in temperature

$$\sigma_{lt} + \sigma_{ll} X_1 = \sigma_{act} \rightarrow \text{zero}$$

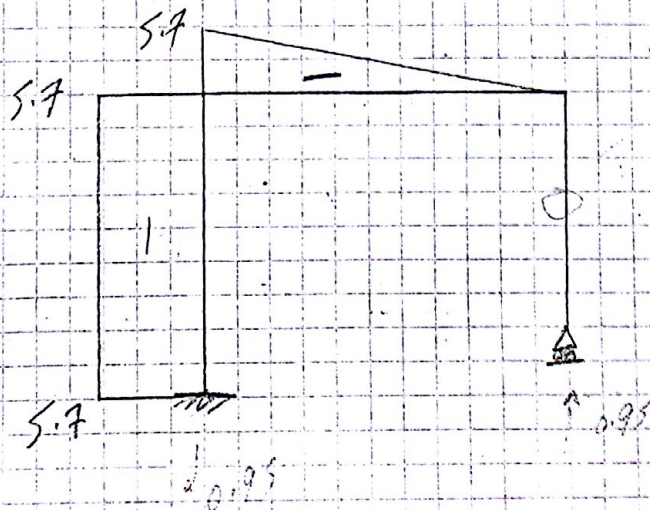
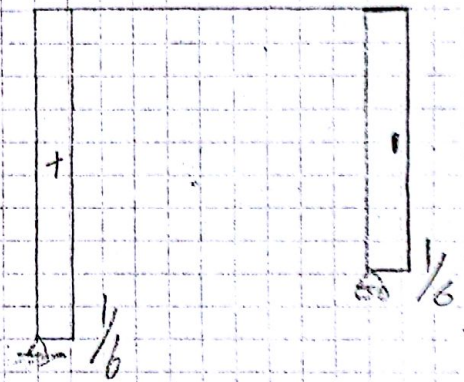
$$\sigma_{lt} = \alpha \left[ \frac{t_{in} - t_{out}}{h} \right] \sum A(M_i) + \frac{t_{in} + t_{out}}{2} \sum A N_i$$

$$\sigma_{lt} = 10^{-5} \times \left[ \frac{40 - 20}{0.8} \right] \times [4 + 6] + \frac{40 + 20}{2} \times \left[ \frac{3}{6} + \frac{4}{6} \right]$$

$$= 2.85 \times 10^{-3}$$

$$2.85 \times 10^{-3} + \frac{1}{2000} X_1 = 0$$

$$X_1 = 5.7 \text{ t.m} \rightarrow$$

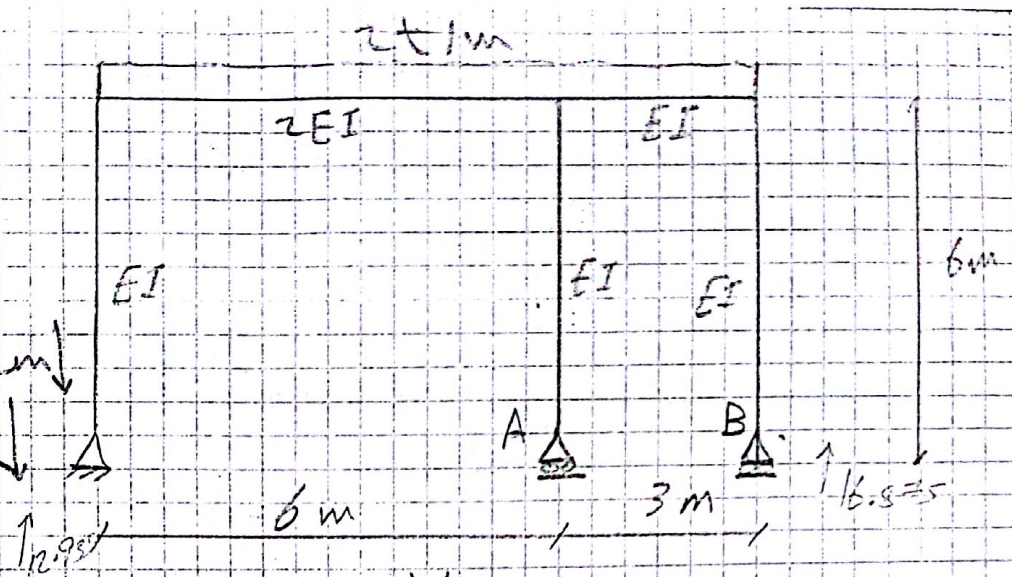




Solve due to shown load

② Settlement at

Support A = 2 cm  
at Support B = 4 cm



$$\delta_{10} = \frac{-3}{8EI} \left[ 2 \times 18 \times 2 \right] - \frac{\frac{2}{3} \times 3 \times \frac{9}{4} \times 1}{EI}$$

$$- \frac{1}{2EI} \left[ 2 \times 2 \times 18 \right] - \frac{\frac{2}{3} \times 6 \times 9 \times 1}{2EI}$$

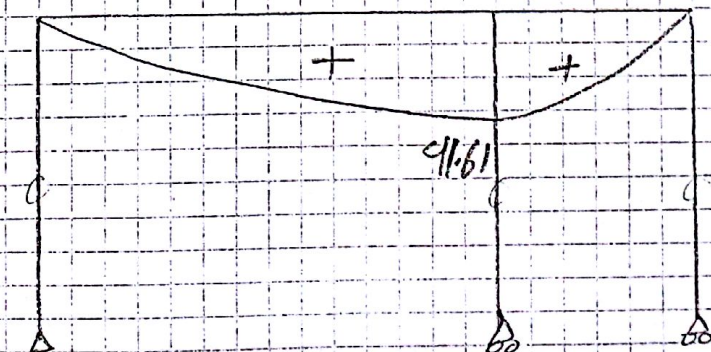
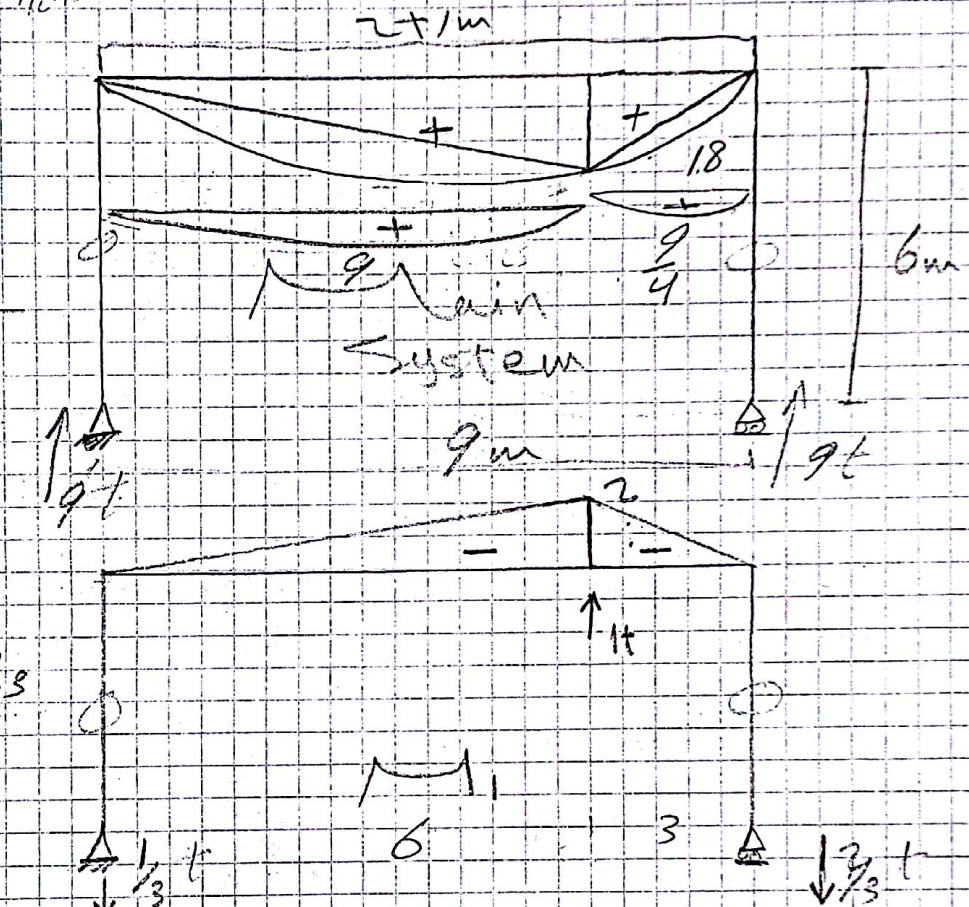
$$\delta_{10} = \frac{-81}{40000} - \frac{54}{20000}$$

$$\delta_{10} = -4.725 \times 10^{-3}$$

$$\delta_{11} = \frac{3}{6EI} \times 2 \times 2 \times 2 + \frac{6}{6EI \times 2} \times 2 \times 2 \times 2$$

$$\delta_{11} = \frac{1}{2500}$$

$$X_1 = - \frac{\delta_{10}}{\delta_{11}} = \frac{-4.725 \times 10^{-3}}{1/2500} = -11.8125 t = 11.81 t \downarrow$$



B.M.D



$$0.15 + X_1 \delta_{11} + \delta_{10} = 0 \text{ act}$$

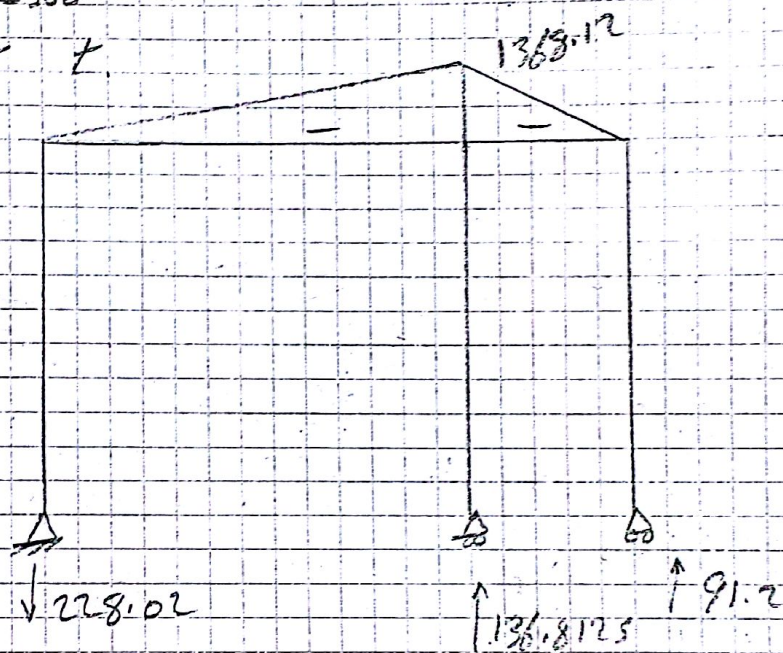
$$\delta_{10} + \sum R \times \text{settlement} = 0$$

$$\delta_{10} + \frac{2}{3} \times 0.04 = 0$$

$$\delta_{10} = -0.03$$

$$-0.03 + X_1 \times \frac{1}{2500} - 4.725 \times 10^{-3} = 0.02$$

$$X_1 = 136.8125 \text{ k}$$



B.M.D

Shown  
Settlement  
at A



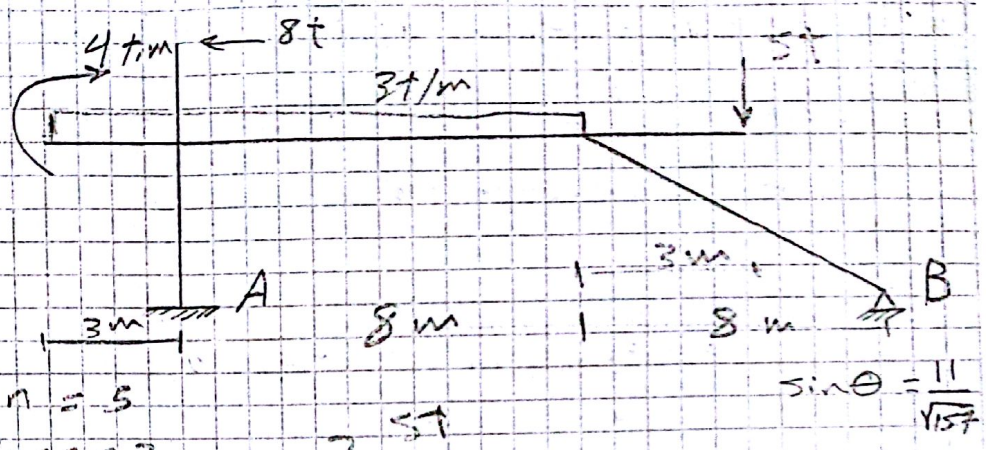
Known load

② Settlement

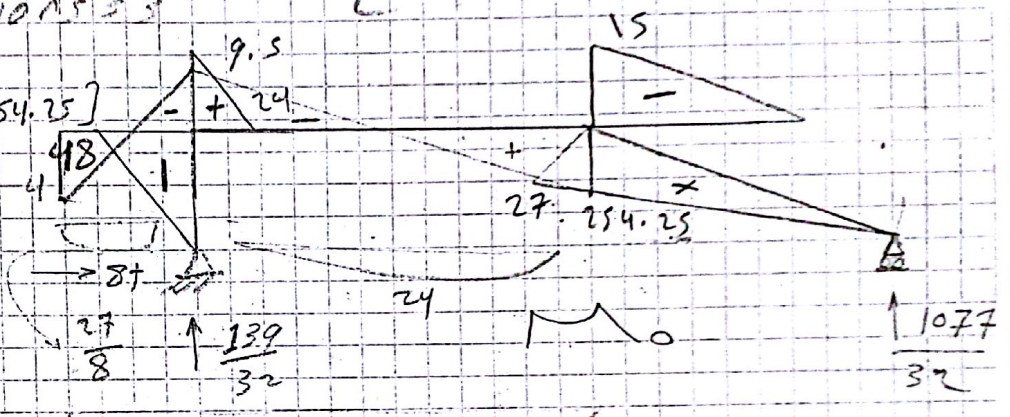
at A = 1 cm ↓

at B = 3 cm ↓

unknown = 5  
Equations = 3

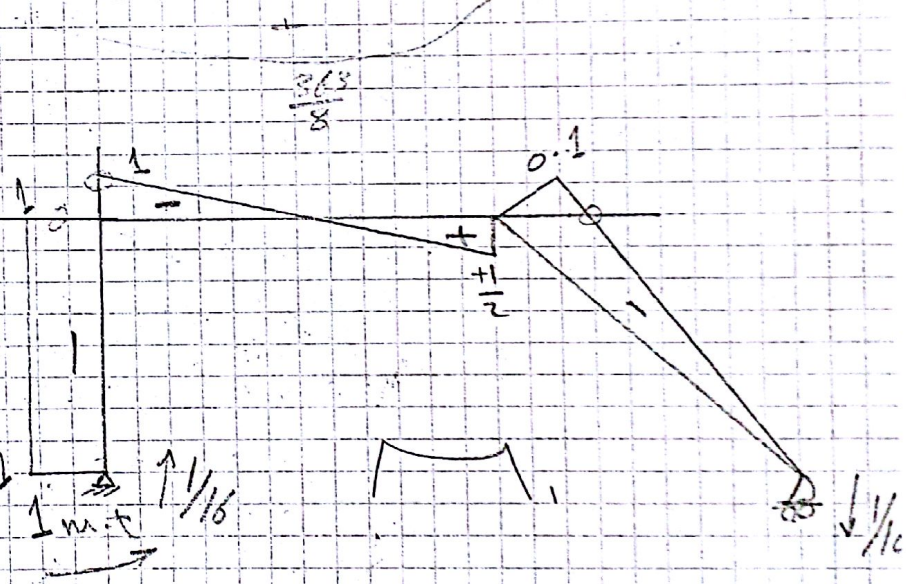


$$\omega_{10} = \frac{8}{6EI} \left[ 2 \times 9.5 \times 1 + \frac{1}{2} \times 254.25 \right] + \frac{6}{6EI} [48 \times 0.5 \times 6] - \frac{10}{6EI} \left[ \frac{27}{8} \times 2 \times 0.1 \right]$$



$$\omega_{10} = 0.025$$

$$\omega_{20} = \frac{6}{6EI} \times [2 \times 48 \times 6] + \frac{10}{6EI} [0.1 \times (-27)] + \frac{8}{6EI} \left[ 2 \times 9.5 \times 1 + \frac{1}{2} \times 254.25 \times \frac{1}{2} \right]$$



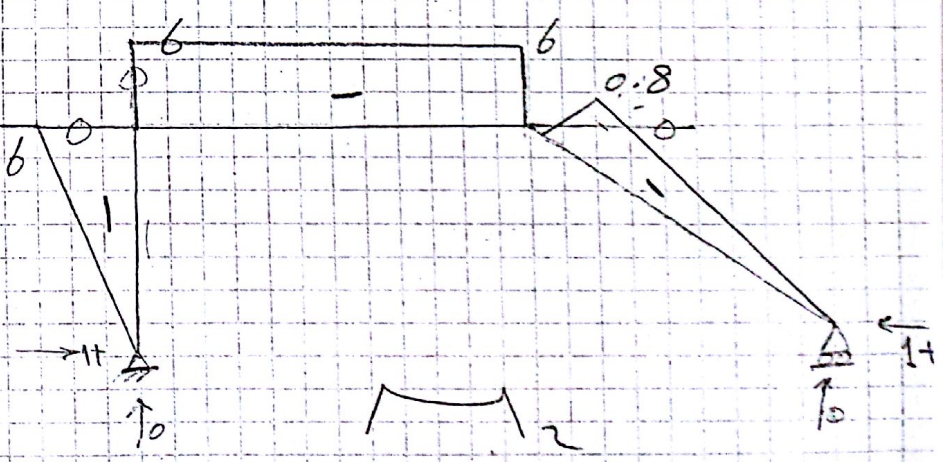
$$\omega_{20} = 0.034$$

$$\omega_{11} = \frac{6}{6EI} \times 6 + \frac{8}{6EI} \left[ 2 \times 1 + \frac{1}{2} \times \frac{1}{2} \right] + \frac{10}{6EI} \times 0.1 \times 2 \times 0.1$$

$$\omega_{11} = 4.7 \times 10^{-4}$$

$$\omega_{22} = \frac{6}{6EI} \times 2 \times 6 \times 6 + \frac{6 \times 8 \times 6}{EI} + \frac{10}{6EI} \times 2 \times 0.8 \times 0.8$$

$$= 0.02$$



$$\Delta_{12} = \frac{6 \times 0.5 \times 6}{EI} + \frac{8}{6EI} \left[ 6 - \frac{1}{2} \times 6 \right] + \frac{0.1 \times 0.8 \times 2 \times 10}{EI} = 1.18 \times 10^{-3}$$



$$a_{10} + X_1 a_{11} + X_2 a_{12} = a_{1det} = 0$$

$$a_{20} + X_1 a_{21} + X_2 a_{22} = a_{2det} = 0$$

$$0.025 + 4.7 \times 10^{-4} X_1 + 1.18 X_2 = 0 \rightarrow \textcircled{1}$$

$$0.034 + 0.02 X_2 + 1.18 X_1 = 0 \rightarrow \textcircled{2}$$

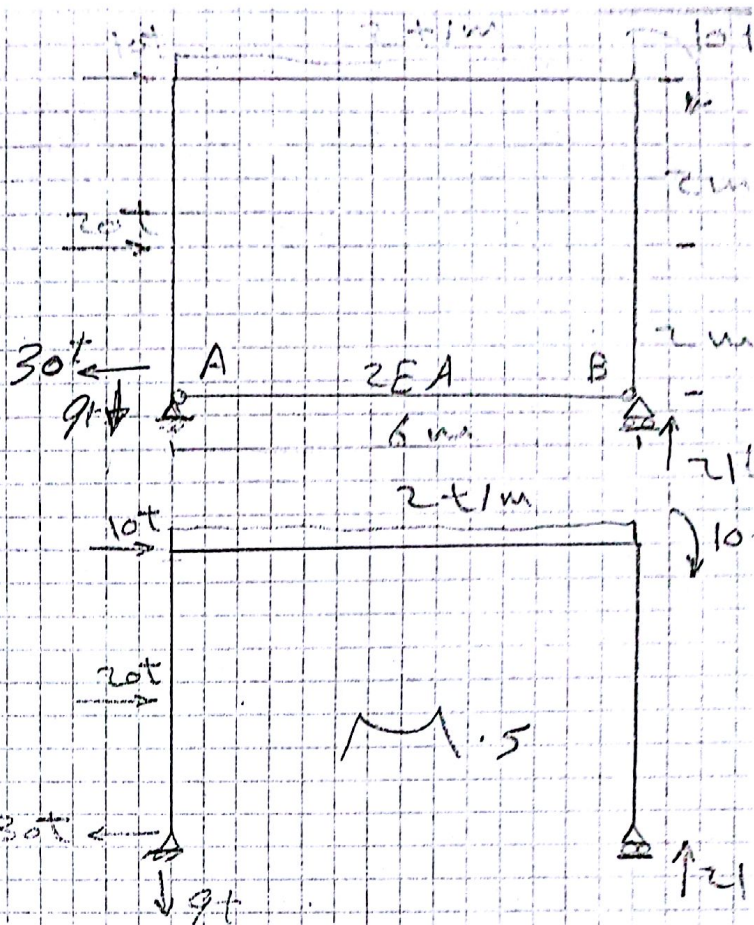
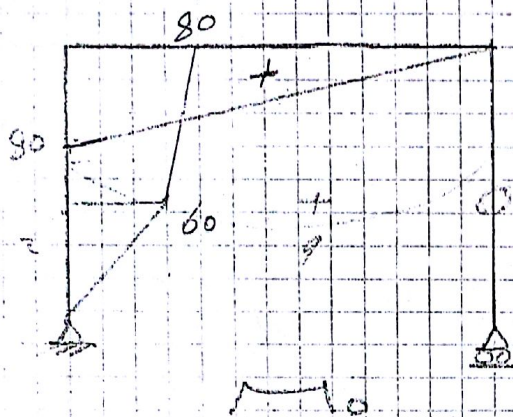
$$X_1 = -0.026$$

$$X_2 = 0.026$$



olve For

- ① Show load
- ② settlement  $A = 3 \text{ cm} \downarrow$   
 $B = 6 \text{ cm} \downarrow$



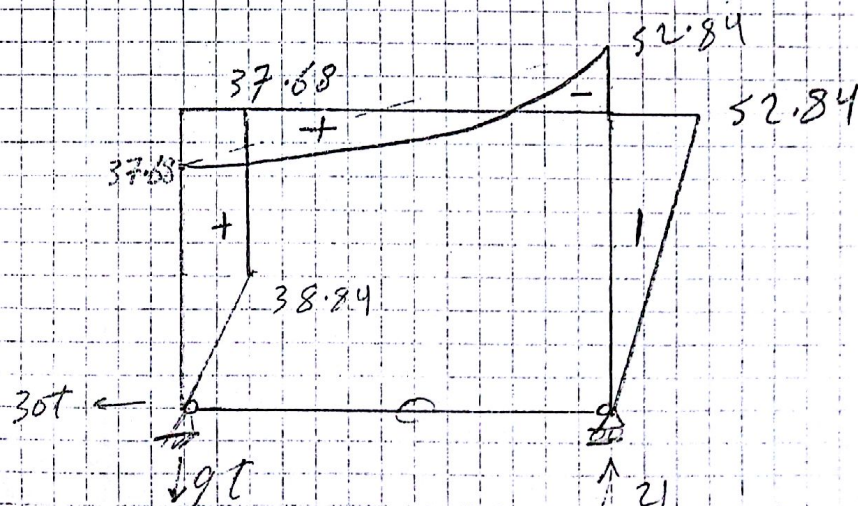
$$\delta_{10} = \frac{0.5 \times 6 \times 80 \times 4}{EI} + \frac{2 \times 2 \times 80 \times 4}{6EI} + \frac{2 \times 60 \times 2}{80 \times 2 + 60 \times 4} + \frac{2 \times 60 \times 2}{6EI}$$

$$\delta_{10} = \frac{1280}{3EI} + \frac{960}{EI} + \frac{80}{EI} = \frac{11}{150}$$

$$\delta_{11} = \frac{4}{6EI} \times 2 \times 4 \times 4 + \frac{4 \times 4 \times 6}{EI} + \frac{4 \times 4 \times 4 \times 2}{6EI} = \frac{13}{1875}$$

$$\delta_{10} + X_1 \delta_{11} = \delta_{1 \text{ act}}$$

$$\frac{11}{150} + X_1 \times \frac{13}{1875} = 0 \quad X_1 = 10.58 \text{ t tension}$$





① Shown loads

② uniform rise in temperature  $40^\circ\text{C}$

③ Settlement at A =  $4\text{cm}$  ↓  
and B =  $6\text{cm}$  ↓

$$\delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0$$

$$\delta_{20} + \delta_{22} X_2 + \delta_{12} X_1 = 0$$

$$\begin{aligned} \delta_{10} &= \frac{+8}{8EI} * [1 * 24 * 4.8] - \frac{2}{8EI} * 8 * 24 * 2.4 \\ &\quad - \frac{2}{8EI} * 6 * \frac{9}{2} * 2.4 \\ &= \frac{-43.2}{EI} \end{aligned}$$

$$\begin{aligned} \delta_{11} &= \frac{8}{8EI} * 2 * 4.8 * 4.8 + \frac{6}{8EI} * 2 * 4.8^2 \\ &= 107.52/EI \end{aligned}$$

$$\delta_{10} + \delta_{11} X_1 = 0$$

$$\frac{-43.2}{EI} + \frac{107.5}{EI} X_1 = 0$$

$$X_1 = 0.4 \text{ t}$$

$$X_1 = 3.04 \text{ t}$$

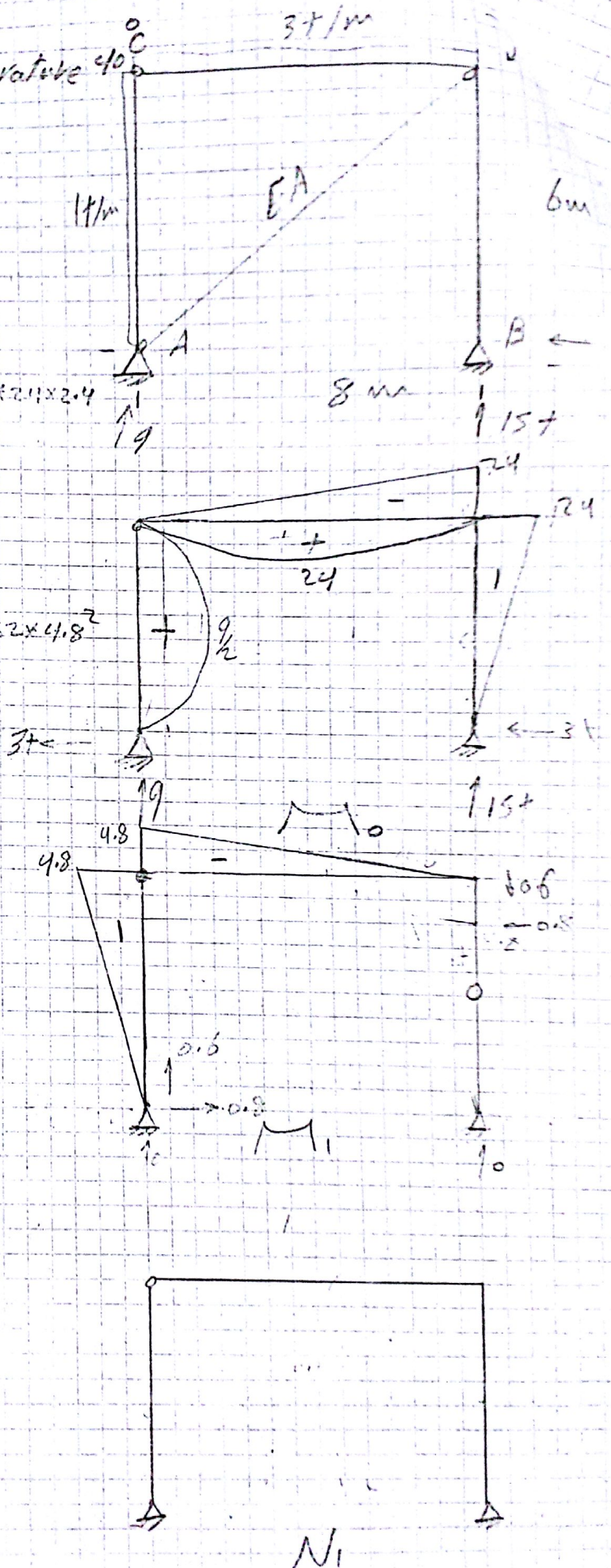
$$\delta_{15} + \delta_{11} X_1 = -0.04$$

$$\delta_{15} + \sum R * \text{sett} = \text{zero}$$

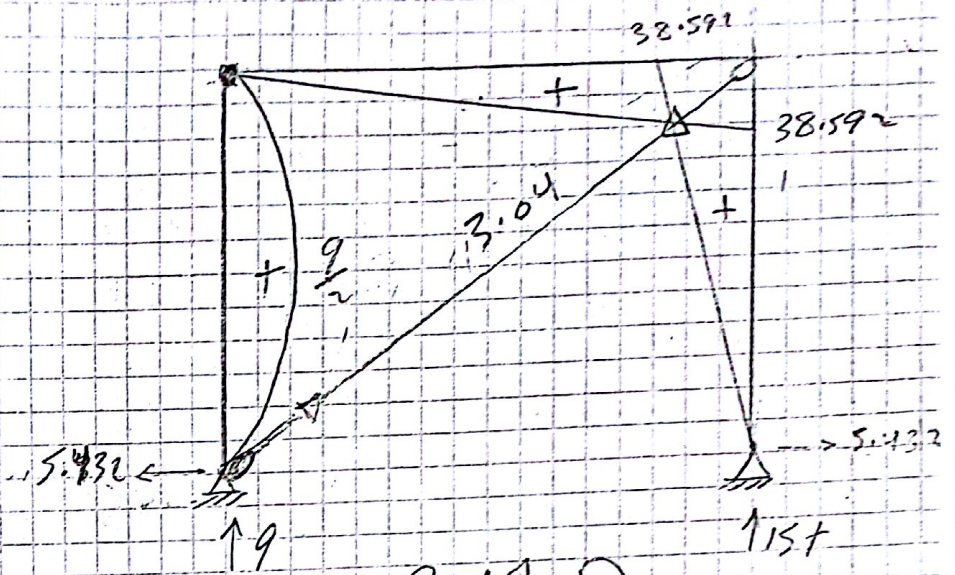
$$\delta_{15} + 0 = 0$$

$$\delta_{11} X_1 = -0.04$$

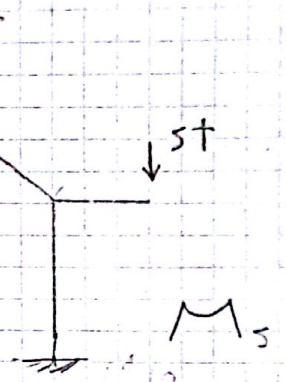
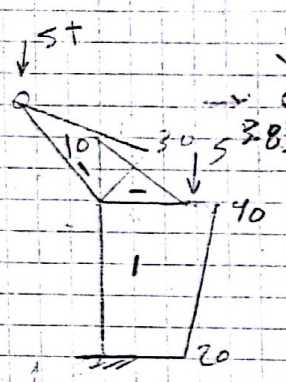
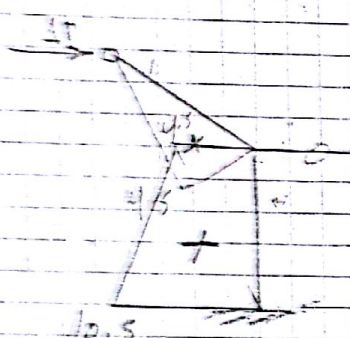
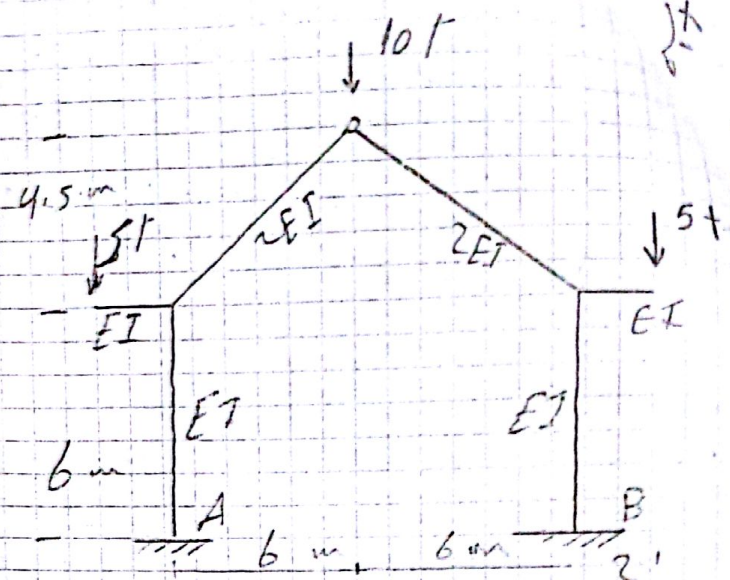
$$X_1 = 0.372 \text{ t}$$











$$M_0 = \frac{-6}{5EI} * [2 * 10.5 * 20 + 2 * 4.5 * 40 + 4.5 * 20 + 10.5 * 40] - \frac{7.5}{2 * 6EI} * 2 * 30 * 4.5$$

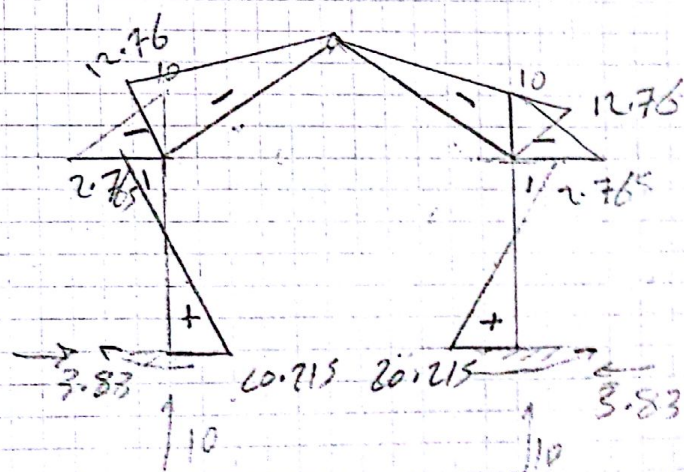
$$= \frac{-1290}{EI} + \frac{-675}{4EI} = \frac{-5835}{4EI}$$

$$\delta_{11} = \frac{7.5}{2 * 6EI} * 2 * 4.5^2 + \frac{6}{6EI} * [2 * 10.5 * 10.5 + 2 * 4.5^2 + (4.5 * 10.5)]$$

$$= \frac{6093}{16EI}$$

$$-\frac{5835}{4EI} + \frac{6093}{16EI} * X_1 = 0$$

$$X_1 = 3.83 \text{ k}$$





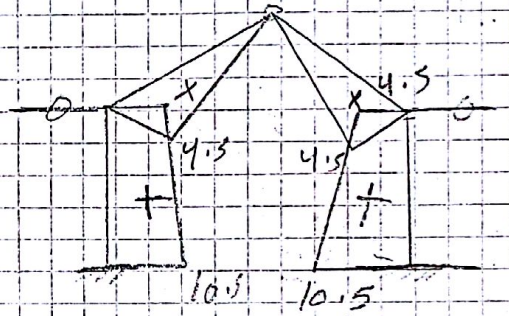
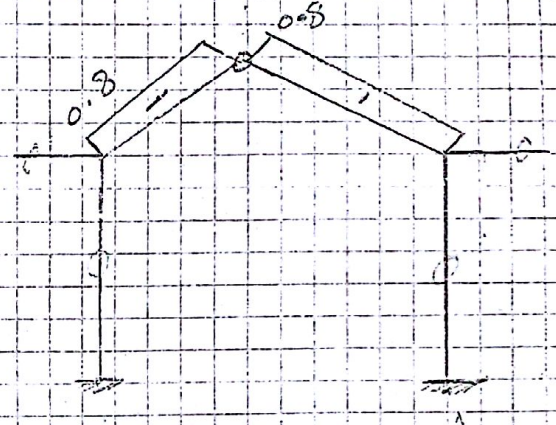
$$\sigma_t = \alpha [t_{in} - t_{out}] \sum A M_i + \alpha \left[ \frac{t_{in} - t_{out}}{2} \right] \sum A N_i$$

$$\sigma_t = 10^{-5} \left[ \frac{40 - 20}{0.2} \right] \times \frac{h}{12} \times (4.5 + 16.9) \times 10^{-5} \times \frac{20}{0.2}$$

$$= 0.0184$$

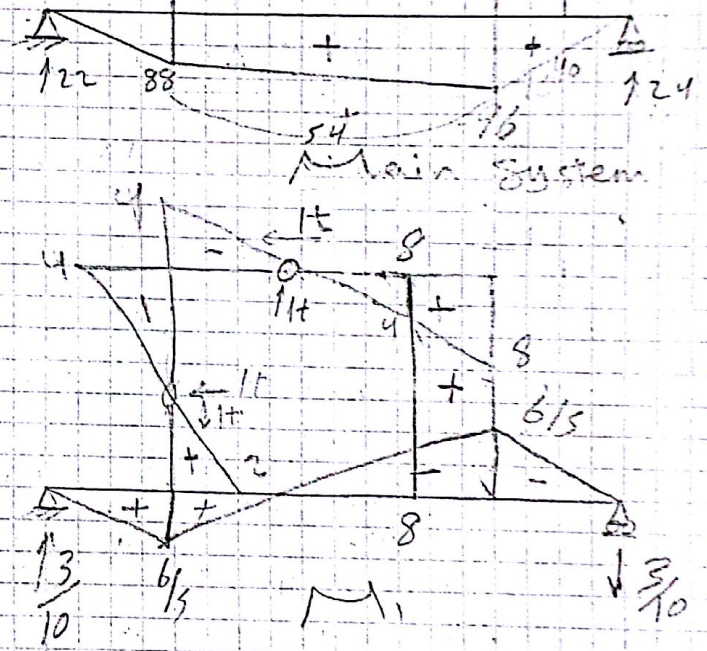
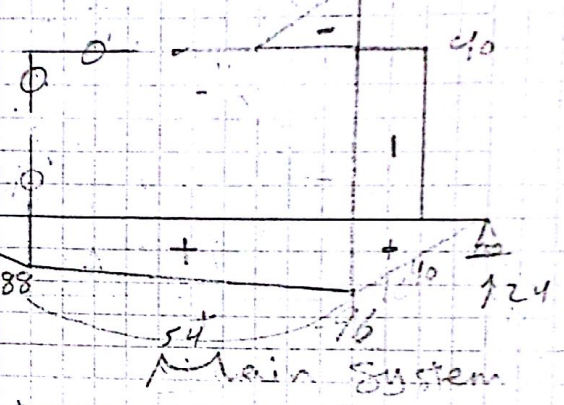
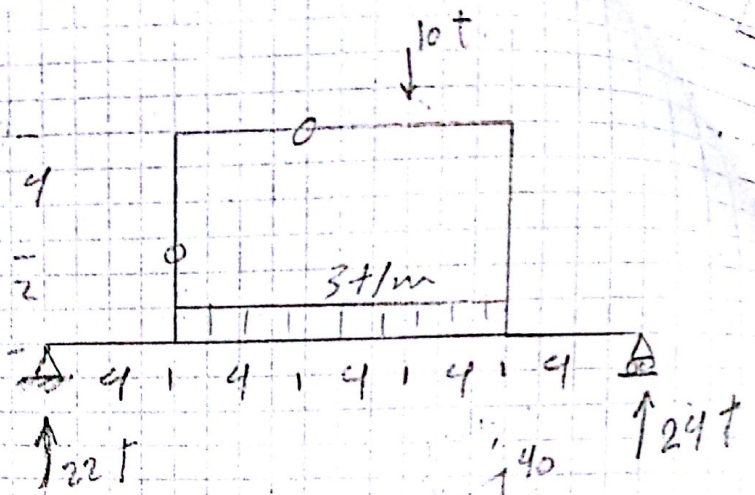
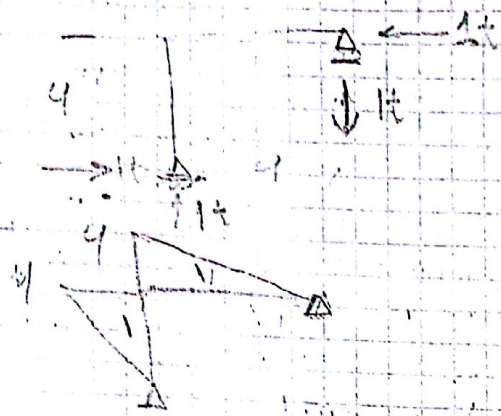
$$\sigma_t + X \cdot \sigma_{t1} = 0$$

$$X = \frac{-0.0184}{2.019} = 1.4$$



B.M.D





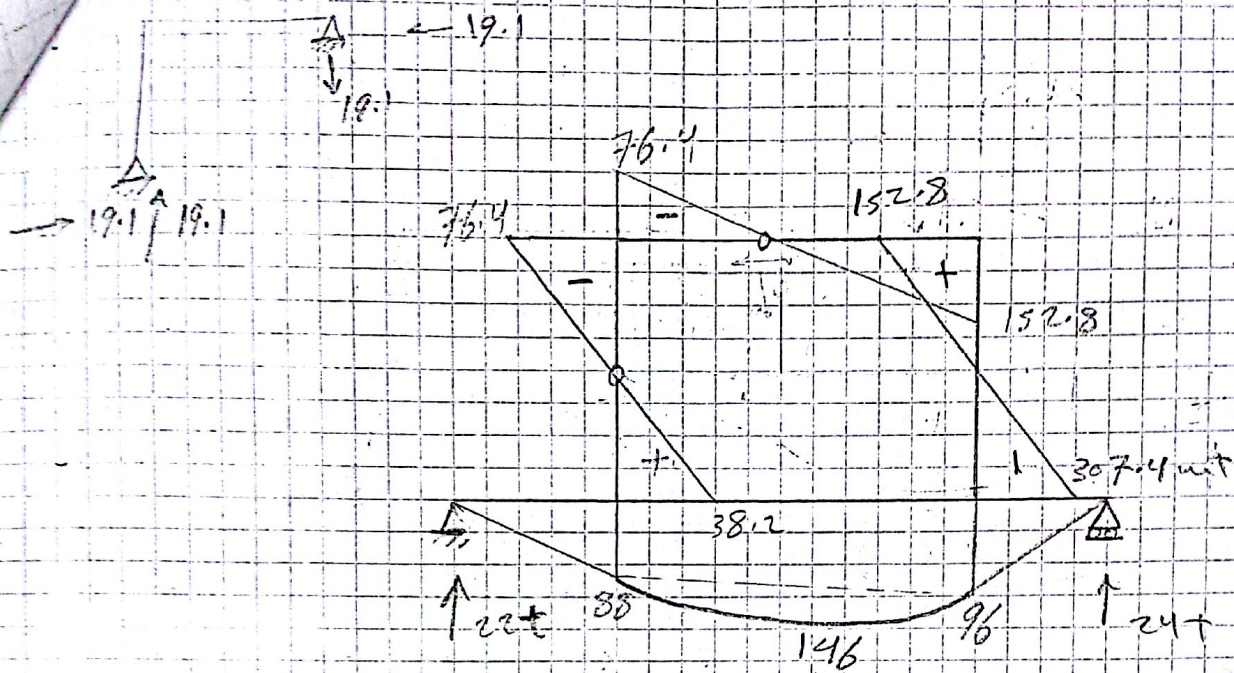
$$\begin{aligned} \delta_{10} &= -\frac{40 \times 6 \times 8}{EI} \\ &- \frac{[2 \times 40 \times 8 + 4 \times 40] \times 4}{6EI} \\ &- \frac{2}{3} \times 54 \times 12 \times \frac{6}{5EI} \\ &+ \frac{88 \times 2 \times \frac{6}{5} \times 4}{6EI} + \frac{2 \times 96 \times \frac{6}{5}}{EI \times 4} \\ &+ \frac{12}{6EI} \times [-2 \times 96 \times \frac{6}{5} + 2 \times 88 \times \frac{6}{5} \\ &- 96 \times \frac{6}{5} - 88 \times \frac{6}{5}] \\ &= -\frac{1920}{EI} - \frac{1600}{3EI} - \frac{2592}{5EI} \\ &+ \frac{768}{5EI} - \frac{288}{5EI} - \frac{240}{EI} \\ &= -\frac{46928}{15EI} \end{aligned}$$

$$\begin{aligned} \delta_{11} &= \frac{12}{6EI} \times [2 \times 4 \times 4 + 2 \times 5 \times 8 - 4 \times 8 \times 2] + \frac{6}{6EI} \times [2 \times 4 \times 4 + 2 \times 2 \times 2 \\ &- 4 \times 2 \times 2] \\ &- \frac{8 \times 6 \times 8}{EI} + \frac{2 \times 4 \times 2 \times \frac{6}{5} \times \frac{6}{5}}{6EI} + \frac{12}{6EI} \times [2 \times \frac{6}{5} \times \frac{6}{5} \times 2 - 2 \times \frac{6}{5} \times \frac{6}{5}] \\ \delta_{11} &= \frac{192}{EI} + \frac{24}{EI} - \frac{384}{EI} + \frac{96}{25EI} = -\frac{4104}{25EI} \end{aligned}$$

$$\delta_{10} + \delta_{11} X_1 = \text{zero}$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = 19.1 \text{ kN} \rightarrow$$





100+